

## Tutorial Week 6

1. Prove that the product of any three consecutive integers is divisible by 6.
2. Show that if  $2^n - 1$  is prime, then  $n$  is prime.  
[*Hint*: Use the identity  $2^{ab} - 1 = (2^a - 1) \cdot (2^{a(b-1)} + 2^{a(b-2)} + \dots + 2^a + 1)$ .]
3. Prove the square root of any prime number is irrational.  
[*Hint*: Proof by contradiction. You may find the following information useful:  
Consider the composite number  $x$  s.t.  $x = w^2 + y^1 + z^1$  where  $w, y, z$  are its factors.  
If one squares  $x$ , one gets  $x^2 = w^4 + y^2 + z^2$ . Note that  $y, z$  occur as even powers.]