Tutorial Week 6

1. Which of the following statements is true?
   $\hookrightarrow \forall x \exists y \; x = 2y + 7$
   $\hookrightarrow \exists x \forall y \; x = 2y + 7$

2. Express each of the statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the words "It is not the case that").

   a) Some dogs can learn new tricks.

   b) No rabbit knows calculus.

   c) Every bird can fly.

   d) There is no dog that can talk.

   e) There is no one in this tutorial who knows Irish and Russian.

3. *Stable Marriage Problem*
   We want to set up a system where two people are married to the "best possible partner". For the purposes of the problem, we'll assume that all marriages are between a single man and a single woman. Each woman indicates her complete preferences over all the men: for each pair of men, which one does she prefer? Similarly, each man indicates his preferences over all the women. A man and a woman introduce an "instability" if they prefer each other to their spouses. A solution to the problem contains no instabilities.

   • Let the set W be the set of women and the set M to be the set of men. We assume that the size (cardinality) of W (written $|W|$) is the same as the size of M.

   • Let $\text{Married}(w, m)$ mean woman w is married to man m ($w \in W, m \in M$).

   (a) Write the predicate $\text{MPoly}(m)$, which means m is married to two (or more) different women.

   (b) We want to require that all men and women in both sets are married and that no one is married more than once. Because the sets are the same size, we can ensure this by saying that each man is married to exactly one woman (no fewer than one and no more than one) and each woman is married to exactly one man. [*Hint*: It will be helpful to use the predicate $\text{MPoly}(m)$!]

(c) • Let C mean the set of integers $1 \ldots |W|$.

• Let WRanks(w, m, n) mean woman w ranks m as her number n choice as a spouse (where n is drawn from C).

Each woman should rank each man with exactly one of the numbers from C. Furthermore, each woman should use each rank available. (i.e., should rank one person for each rank)
Together with the fact that C contains exactly as many elements as $M(|C| = |M|$, this indicates that no woman ever reuses a rank.)
Write this condition in predicate logic. [*Hint*: It will be useful to write a helper predicate first!]

(d) Define the predicate WPrefers(w, m_1, m_2) to mean woman w prefers m_1 to m_2; that is, woman w ranks m_1 with a lower number (closer to 1) than m_2.
(Because of the condition you established in the previous part, this will automatically mean that every woman has a clear preference in every comparison.)

(e) Define a new predicate Instability(w, m). This is true if w and m cause an instability, as described above. Use WPrefers from the previous part, and assume you have a corresponding predicate MPrefers describing mens preferences and Married

(f) Using your predicate from the previous part, write the condition that there is no instability.