Tutorial Week 5

Last updated January 27th, 2012

1. Design a circuit that works with two 4-bit numbers and outputs true if either:
   the two numbers are equal
   OR
   the numbers differ by 8 *and* are both odd
   (This question is similar to 2012W2 a2 q2)

Following questions are from Harry Rosen sec 1.3

2. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language Malbolge (*mah-leh-bol-djeh*)." Express each of these sentences in terms of $P(x), Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

   a) There is a student at your school who can speak Russian and who knows Malbolge.

   b) There is a student at your school who can speak Russian but doesn't know Malbolge.

   c) Every student at your school either can speak Russian or knows Malbolge.

   d) No student at your school can speak Russian or knows Malbolge.

3. Express each of these statements using logical operators, predicates and quantifiers:

   a) Some propositions are tautologies.

   b) The negation of a contradiction is a tautology.

   c) The disjunction(OR) of two contingencies(statements) can be a tautology.

   d) The conjunction(AND) of two tautologies is a tautology.

Following questions are from Harry Rosen sec 1.4

4. Let $L(x, y)$ be the statement "x loves y", where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements:

   a) Everybody loves Étienne.

   b) Nobody loves everybody.

   c) There is somebody whom Albert does not love.

d) There is exactly one person whom everybody loves.

e) There are exactly two people whom Natasha loves.

f) Everyone loves himself or herself.

g) There is someone who loves no one besides himself or herself.

5. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

   a) $\forall n \exists m(n^2 < m)$

   b) $\exists n \forall m(nm = m)$

   c) $\exists n \exists m(n^2 + m^2 = 6)$

   d) $\exists n \exists m(n + m = 4 \wedge n - m = 2)$

   e) $\forall n \forall m \exists p(p = (m + n)/2)$