9. Let $L(x, y)$ be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody loves Jerry.

b) Everybody loves somebody.

c) There is somebody whom everybody loves.

d) Nobody loves everybody.

e) There is somebody whom Lydia does not love.

f) There is somebody whom no one loves.

g) There is exactly one person whom everybody loves.

h) There are exactly two people whom Lynn loves.

i) Everyone loves himself or herself.

j) There is someone who loves no one besides himself or herself.

both do not like it). **9. a)** $\forall x L(x, \text{Jerry})$ **b)** $\forall x \exists y L(x, y)$ **c)** $\exists y \forall x L(x, y)$ **d)** $\forall x \exists y \neg L(x, y)$ **e)** $\exists x \neg L(\text{Lydia}, x)$ **f)** $\exists x \forall y \neg L(y, x)$ **g)** $\exists x (\forall y L(y, x) \wedge \forall z ((\forall w L(w, z)) \rightarrow z = x))$ **h)** $\exists x \exists y (x \neq y \wedge L(\text{Lynn}, x) \wedge L(\text{Lynn}, y) \wedge \forall z (L(\text{Lynn}, z) \rightarrow (z = x \vee z = y)))$ **i)** $\forall x L(x, x)$ **j)** $\exists x \forall y (L(x, y) \leftrightarrow x = y)$ **11. a)** $A(\text{Lois, Professor Michaels})$

27. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a) $\forall n \exists m (n^2 < m)$

b) $\exists n \forall m (n < m^2)$

c) $\forall n \exists m (n + m = 0)$

d) $\exists n \forall m (nm = m)$

e) $\exists n \exists m (n^2 + m^2 = 5)$

f) $\exists n \exists m (n^2 + m^2 = 6)$

g) $\exists n \exists m (n + m = 4 \land n - m = 1)$

h) $\exists n \exists m (n + m = 4 \land n - m = 2)$

i) $\forall n \forall m \exists p (p = (m + n)/2)$

the operation of addition. **27. a)** True **b)** True **c)** True **d)** True **e)** True **f)** False **g)** False **h)** True **i)** False

33. Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

a) $\neg\forall x \forall y P(x, y)$

b) $\neg\forall y \exists x P(x, y)$

c) $\neg\forall y \forall x (P(x, y) \vee Q(x, y))$

d) $\neg(\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$

e) $\neg\forall x (\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$

33. a) $\exists x \exists y \neg P(x, y)$ **b)** $\exists y \forall x \neg P(x, y)$ **c)** $\exists y \exists x (\neg P(x, y) \wedge \neg Q(x, y))$ **d)** $(\forall x \forall y P(x, y)) \vee (\exists x \exists y \neg Q(x, y))$ **e)** $\exists x (\forall y \exists z \neg P(x, y, z) \vee \forall z \exists y \neg P(x, y, z))$ **35.** Any domain

39. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x \forall y(x^2 = y^2 \rightarrow x = y)$

b) $\forall x \exists y(y^2 = x)$

c) $\forall x \forall y(xy \geq x)$

with Kevin Bacon. **39. a)** $x = 2, \quad y = -2$ **b)** $x = -4$ **c)** $x = 17, y = -1$ **41.** $\forall x \, \forall y \, \forall z ((x \cdot y) \cdot z = x \cdot (y \cdot z))$