Tutorial Week 2

Last updated January 21st, 2012

1. Make circuit for the following Truth Table: Can you simplify it?

Truth Table			
p	q	r	output
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

First approach maybe:

$$(\sim p \lor \sim q \lor \sim r) \land (\sim p \lor \sim q \lor r) \land (\sim p \lor q \lor \sim r) \land (p \lor \sim q \lor r)$$

Simplify (**hints** : the brute force approach needs three variables to describe each individual time the output is 1. is there a way to use less than three variables to cover more than 1 case where the output is 1?)

One simplify answer: observe that the rule $(\sim p \land \sim r)$ covers rows 0 and 2 and the rule $(\sim q \land r)$ covers the other two rows where the circuit is true (1 and 5)

$$(\sim p \land \sim r) \lor (\sim q \land r)$$

with deMorgan's law

$$(\sim p \lor r) \lor (\sim q \land r)$$

Another simplified answer: Observe that the first three rows are true, which is $\sim p$ + the opposite of $(q \land r)$, then covering the last row the brute force way

$$(\sim p \land \sim((q \land r)) \lor (p \land \sim q \land r)$$

2. Prove the logical equivalence below:
$\sim p \equiv (\sim p \vee s) \wedge (p \to (s \to \sim p))$

$$\begin{aligned}
\sim p &\equiv (\sim p \vee s) \wedge (\sim p \vee (\sim s \vee \sim p)) && \text{by definition of conditional} \\
\sim p &\equiv (\sim p \vee s) \wedge ((\sim p \vee \sim p) \vee \sim s) && \text{by associative law} \\
\sim p &\equiv (\sim p \vee s) \wedge (\sim p \vee \sim s) && \text{by idempotent law} \\
\sim p &\equiv \sim p \vee (s \wedge \sim s) && \text{by distributive law} \\
\sim p &\equiv \sim p \vee F && \text{by negation law} \\
\sim p &\equiv \sim p && \text{by identity law}
\end{aligned}$$

3. Represent 1609_{10} in binary

$$\begin{aligned}
1609_{10} &= 1024 + 512 + 64 + 8 + 1 \\
&= 1 \cdot 2^{10} + 1 \cdot 2^{9} + 1 \cdot 2^{6} + 1 \cdot 2^{3} + 1 \cdot 2^{0} \\
&= 11001001001_2
\end{aligned}$$

4. Represent 11000111_2 in decimal

$$\begin{aligned}
11000111_2 &= 1 \cdot 2^{7} + 1 \cdot 2^{6} + 1 \cdot 2^{2} + 1 \cdot 2^{1} + 1 \cdot 2^{0} \\
&= 128 + 64 + 4 + 2 + 1 \\
&= 199
\end{aligned}$$

5. Find the 2's complement of 410 in 16 bit, i.e. 410_16 or 410 as a hex number.

$$\begin{aligned}
410_8 &= 0100\ 0001\ 0000 \\
&\Rightarrow 101111101111 \\
&\Rightarrow 101111110000
\end{aligned}$$

6. Find the signed and unsigned decimal representation of 11110010
Signed:

$$\begin{aligned}
11110010_2 &\Rightarrow 11110010 + 1 \\
&\Rightarrow 11110011 \\
&\Rightarrow 00001100 \\
&= 1 \cdot 2^{3} + 1 \cdot 2^{2} \\
&= 8 + 4 \\
&= 12
\end{aligned}$$

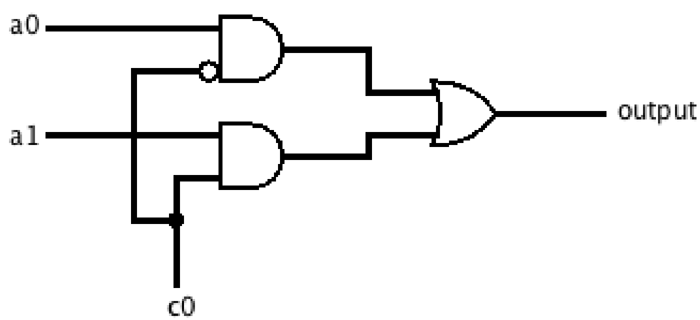
Unsigned:

$$\begin{aligned} 11110010_2 &= 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^1 \\ &= 128 + 64 + 32 + 16 + 2 \\ &= 242 \end{aligned}$$

7. BCD represents k decimal digits using 4k bits in groups of 4. Each group of 4 represents a single digit (0-9). So, for example, 59 would be 01011001 in BCD, a 5 (0101) followed by a 9 (1001).

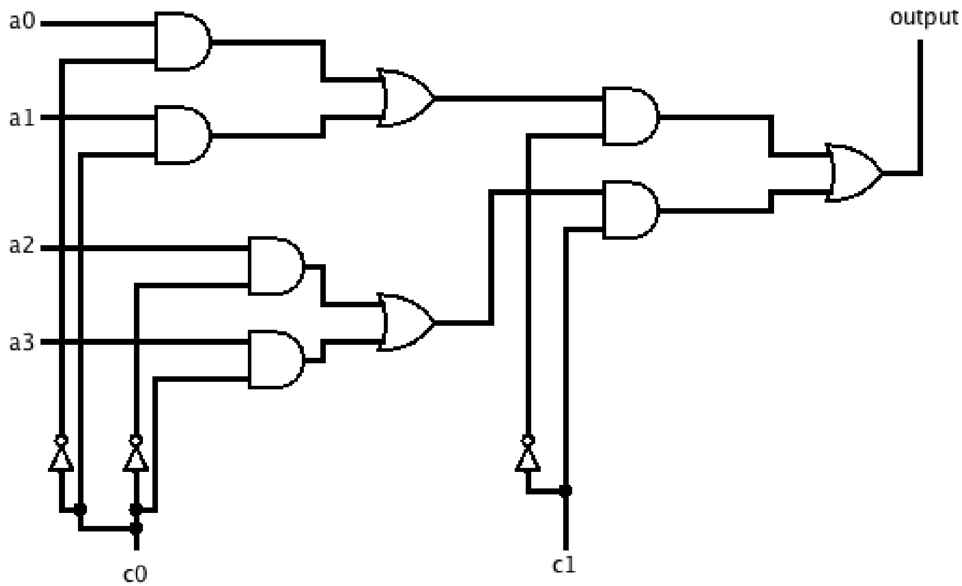
8. Design the logic statements for a circuit that adds 5 to a 1-digit (4-bit) BCD number $i_1 i_2 i_3 i_4$. **Assume the input value is 4 or less**; so, the output is 9 or less. The rightmost bit of the output $o_4 = \sim i4$. The second ...

9. Consider the following multiplexer design (delay issue is ignored):

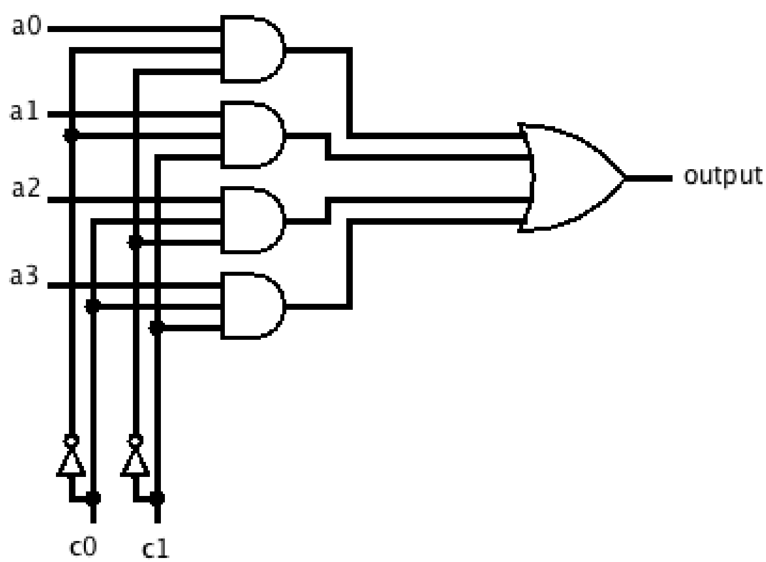


Provide 2 circuits that extend this design to be 4-input.

Hint, modular use of first plexer:



Hint, 3-input and 4-input gates:



10. Extra: Prove that $(p \vee q) \equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee ((p \wedge \sim p) \wedge (q \vee \sim q))$

$\equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee ((p \wedge \sim p) \wedge (q \vee \sim q))$	
$\equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee ((p \wedge \sim p) \wedge T)$	by negation
$\equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee (F \wedge T)$	by negation
$\equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee F$	by universal bound
$\equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q)$	by identity
$\equiv (\sim q \to p) \vee (p \vee q)$	by De Morgan's
$\equiv (\sim p \to q) \vee (p \vee q)$	by contrapositive
$\equiv (p \vee q) \vee (p \vee q)$	by definition of conditional
$\equiv (p \vee q)$	by Idempotent