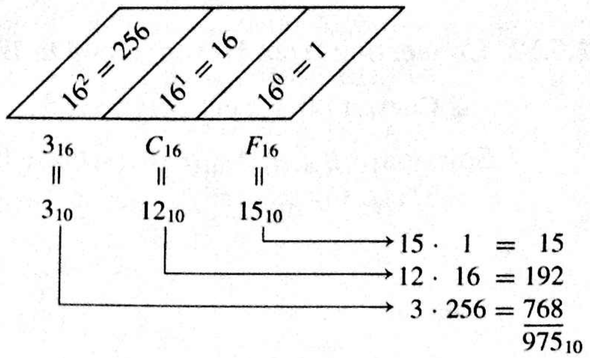
Example 1.5.11 Converting from Hexadecimal to Decimal Notation

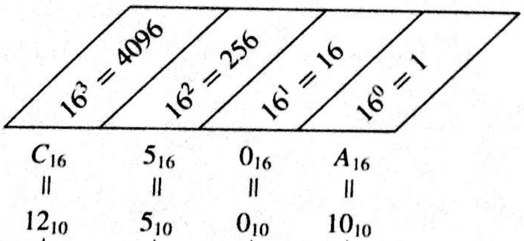
Convert $3CF_{16}$ to decimal notation.

Solution A schema similar to the one introduced in Example 1.5.2 can be used here.



So $3CF_{16} = 975_{10}$. ■

Now consider how to convert from hexadecimal to binary notation. In the example below the numbers are rewritten using powers of 2, and the laws of exponents are applied. The result suggests a general procedure.



$10 \cdot 16^0 = (2^3 + 2) \cdot 1 \quad = 2^3 + 2$ since $10 = 2^3 + 2$

$0 \cdot 16^1 = 0 \cdot 2^4 \quad = 0$ since $16^1 = 2^4$

$5 \cdot 16^2 = (2^2 + 1) \cdot 2^8 \quad = 2^{10} + 2^8$ since $5 = 2^2 + 1$, $16^2 = (2^4)^2 = 8$ and $2^2 \cdot 2^8 = 2^{10}$

$12 \cdot 16^3 = (2^3 + 2^2) \cdot 2^{12} = 2^{15} + 2^{14}$ since $12 = 2^3 + 2^2$, $16^2 = (2^4)^3 = 2^{12}$, $2^3 \cdot 2^{12} = 2^{15}$, and $2^2 \cdot 2^{12} = 2^{14}$

But

$$(2^{15} + 2^{14}) + (2^{10} + 2^8) + 0 + (2^3 + 2)$$

$$= 1100\ 0000\ 0000\ 0000_2 + 0101\ 0000\ 0000_2 + 0000\ 0000_2 + 1010_2$$ by the rules for writing binary numbers.

So

$$C50A_{16} = \underbrace{1100}_{C_{16}}\ \underbrace{0101}_{5_{16}}\ \underbrace{0000}_{0_{16}}\ \underbrace{1010_2}_{A_{16}}$$ by the rules for adding binary numbers.

The procedure illustrated in this example can be generalized. In fact, the following sequence of steps will always give the correct answer:

Step 3: Find the decimal equivalent of the result. Because its leading bit is 1, this number is the 8-bit representation of a negative integer.

$$10001110 \xrightarrow{\text{flip the bits}} 01110001 \xrightarrow{\text{add 1}} 01110010_2$$
$$\leftrightarrow -(64+32+16+2)_{10} = -114_{10}$$

■

Since $(-89) + (-25) = -114$, that is the correct answer.

Hexadecimal Notation

It should now be obvious that numbers written in binary notation take up much more space than numbers written in decimal notation. Yet many aspects of computer operation can best be analyzed using binary numbers. **Hexadecimal notation** is even more compact than decimal notation, and it is much easier to convert back and forth between hexadecimal and binary notation than it is between binary and decimal notation. The word *hexadecimal* comes from the Greek root *hex*-, meaning "six," and the Latin root *deci*-, meaning "ten." Hence *hexadecimal* refers to "sixteen," and hexadecimal notation is also called **base 16 notation.** Hexadecimal notation is based on the fact that any integer can be uniquely expressed as a sum of numbers of the form

$$d \cdot 16^n,$$

where each n is a nonnegative integer and each d is one of the integers from 0 to 15. In order to avoid ambiguity, each hexadecimal digit must be represented by a single symbol. So digits 10 through 15 are represented by the first six letters of the alphabet. The sixteen hexadecimal digits are shown in Table 1.5.3, together with their decimal equivalents and, for future reference, their 4-bit binary equivalents.

Table 1.5.3

Decimal	Hexadecimal	4-Bit Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

> To convert an integer from hexadecimal to binary notation:
>
> - Write each hexadecimal digit of the integer in fixed 4-bit binary notation.
> - Juxtapose the results.

Example 1.5.12 Converting from Hexadecimal to Binary Notation

Convert $B09F_{16}$ to binary notation.

Solution $B_{16} = 11_{10} = 1011_2$, $0_{16} = 0_{10} = 0000_2$, $9_{16} = 9_{10} = 1001_2$, and $F_{16} = 15_{10} = 1111_2$. Consequently,

$$\begin{array}{cccc} B & 0 & 9 & F \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 1011 & 0000 & 1001 & 1111 \end{array}$$

and the answer is 1011000010011111_2. ■

To convert integers written in binary notation into hexadecimal notation, reverse the steps of the previous procedure.

> To convert an integer from binary to hexadecimal notation:
>
> - Group the digits of the binary number into sets of four, starting from the right and adding leading zeros as needed.
> - Convert the binary numbers in each set of four into hexadecimal digits. Juxtapose those hexadecimal digits.

Example 1.5.13 Converting from Binary to Hexadecimal Notation

Convert 100110110101001_2 to hexadecimal notation.

Solution First group the binary digits in sets of four, working from right to left and adding leading 0's if necessary.

$$0100 \quad 1101 \quad 1010 \quad 1001.$$

Convert each group of four binary digits into a hexadecimal digit.

$$\begin{array}{cccc} 0100 & 1101 & 1010 & 1001 \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 4 & D & A & 9 \end{array}$$

Then juxtapose the hexadecimal digits.

$$4DA9_{16}$$

■

Example 1.5.14 Reading a Memory Dump

The smallest addressable memory unit on most computers is one byte, or eight bits. In some debugging operations a dump is made of memory contents; that is, the contents