

Midterm Review  
Feb 5th, 2012

## 1 Circuits and Truth Tables

1. Create a circuit for the following truth table:

First we will consider the truth table values that result in the output being 1.

Table Values				
p	q	r	s	output
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

 $\Rightarrow$ 

True Values				
p	q	r	s	output
0	0	0	0	1
0	0	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	0	1
1	1	1	1	1

Now we have a table we can build the circuit from.

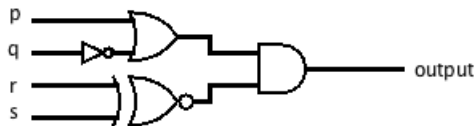
Looking at  $r$  and  $s$  we can see that the output is true when the values are the same, i.e. an XNOR. *Note:* We cannot simply write a circuit as based on an XNOR of  $r$  and  $s$  values because there are other cases where this does not hold in the original table. We must be conscious of the original table in constructing our circuit.

However, we know the XNOR is crucial so half our circuit will be based on this:

$$(p \vee \sim q) \wedge \sim(r \oplus s)$$

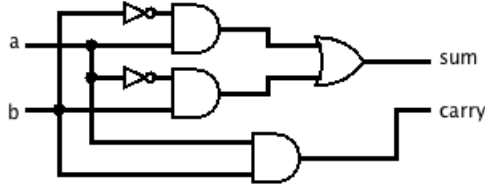
Next, looking at the columns of  $p$  and  $q$  it appears that the smaller table highlights any case where either  $p$  is true or  $q$  is false. Checking the main table, this seems to be okay. So our circuit is  $(p \vee \sim q) \wedge \sim(r \oplus s)$

And finally, we can build our circuit:



2. Give the formula, the truth table, and draw the circuit for the sum and carry outputs of a binary half-adder, that is, a circuit with which we can add two 1-bit numbers.

$$\begin{aligned} \text{sum} &\equiv (a \wedge \sim b) \vee (\sim a \wedge b) \\ \text{carry} &\equiv a \wedge b \end{aligned}$$



## 2 Logical proofs

*Note:* There may well be much faster solutions than what is written down. As long as you arrive at the correct solution without tons of roundabout steps, you will get full marks for a question. If the proof is not too long, often you will get a hint such as “The proof to this is 9 lines. You may not get exactly this, but keep it in mind as a ballpark.”

1. An exclusive or gate is true only when one but not both of its inputs is on. An exclusive nor gate is true when both inputs are the same, either both on or both off. To convert an XOR gate to an XNOR gate one simply adds a hex inverter. Prove that the hex inverter can be placed at either one of the two inputs or the output and still achieve the same result.

Essentially you are being asked to prove that  $\sim(p \oplus q) \equiv (\sim p \oplus q) \equiv (p \oplus \sim q)$  where  $p \oplus q \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

Case  $(\sim p \oplus q) \equiv \sim(p \oplus q)$

$$\begin{aligned} (p \wedge q) \vee (\sim p \wedge \sim q) &\equiv \sim((p \wedge \sim q) \vee (\sim p \wedge q)) && \\ &\equiv \sim(p \wedge \sim q) \wedge \sim(\sim p \wedge q) && \text{By DeMorgan's} \\ &\equiv (\sim p \vee q) \wedge (p \vee \sim q) && \text{By DeMorgan's} \\ &\equiv ((\sim p \vee q) \wedge p) \vee ((\sim p \vee q) \wedge \sim q) && \text{By Distributive} \\ &\equiv ((\sim p \wedge p) \vee (q \wedge p)) \vee ((\sim p \wedge \sim q) \vee (q \wedge \sim q)) && \text{By Distributive} \\ &\equiv ((F \vee (q \wedge p)) \vee ((\sim p \wedge \sim q) \vee F)) && \text{By Negation} \\ &\equiv (p \wedge q) \vee (\sim p \wedge \sim q) && \text{By Identity, Commutative} \end{aligned}$$

The proof for the second case (i.e.  $p \oplus \sim q$ ) is exactly the same, except we substitute p for q. By Commutative, all the steps and results will be identical.

2. Prove  $u \wedge w$  using a formal propositional logic proof given premises 1-7. (*No Solution*)

1.  $p \rightarrow q$
2.  $r \vee s$
3.  $\sim s \rightarrow \sim t$
4.  $\sim q \vee s$
5.  $\sim s$
6.  $\sim p \wedge r \rightarrow u$
7.  $w \vee t$

### 3 Additional Relevant Questions

The following questions I chose as being particularly useful in what was missing from this sheet. Note I have not included some topics, but this is not to say they are not on the midterm ***Especiallly*** don't forget predicates, quantifiers etc.

- 2006/7 Winter Term 2                      Good proof question:  $(p \oplus q) \vee (p \wedge q) \vee r \equiv p \vee (q \vee r)$
- 2009W1 Part 2, Part 5-2.                  Unusual propositional logic question, more difficult proof
- 2011S Question 3 is kindof fun :)        Understanding some made-up laws
- 2010W2 Q3, Q6                              Propositional translation, Circuits
- 2007/8W1 Question 4                      A harder midterm question about number representation and circuits